SAE TECHNICAL PAPER SERIES

2001-01-1283

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Reprinted From: Accident Reconstruction-Crash Analysis (SP–1572)

The Engineering Society For Advancing Mobility Land Sea Air and Space®
INTERNATIONAL

SAE 2001 World Congress
Detroit, Michigan
March 5-8, 2001

400 Commonwealth Drive, Warrendale, PA 15096-0001 U.S.A. Tel: (724) 776-4841 Fax: (724) 776-5760

ISSN 0148-7191

Printed in USA

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ABSTRACT

This paper presents various reconstruction methods for a multiple-car-collision, resulting from one vehicle approaching a line of stationary, undamaged vehicles that consequently pushes the stationary vehicles into one another. Under these conditions, the approaching vehicle speed and delta-V of all vehicles can be estimated from the summation of all vehicles' front and rear damage plus their run-out energies. However, seldom are all vehicles available for damage inspection, or are the skid/gouge-marks or friction-coefficients adequately known. The accident severity of the collision pair of interest would be reconstructed more efficiently if the damage energy method were applied to only this collision pair. This method, however, entails the concern that the observed damage of these two vehicles has been enhanced by the subsequent collisions.

An analysis is presented to specify the conditions under which initial vehicle damage of a collision pair is not enhanced by successive collisions, such that the initial delta-V of the vehicles of interest can be calculated using only the front and rear damage energy of that pair. The vehicles must have low restitution and sufficient space between them, such that the vehicle pile-up can be treated like a sequence of two-vehicle crashes (where the approaching vehicle may consist of a combination of the previously collided vehicles). The method is applicable for most real-world accidents, unless one or more of the vehicles is significantly heavier and/or stiffer than the vehicles of interest.

INTRODUCTION

Accident reconstruction methods are often applied to single-vehicle or vehicle-to-vehicle collisions. Damage analysis and run-out analysis are both tools that apply the principles of energy and momentum conservation to estimate the impact energy and accident severity or delta-V for either of the vehicles of interest.

The damage energy method builds on the comparison of the crush of the collided vehicles with the crush in corresponding, controlled crash tests performed by government and industry. The crash tests have both the vehicle masses, speeds and residual crush well documented, such that the pre-crash kinetic energy can be related to the crush amount through crush models like the linear stiffness model (Campbell, 1974), the force saturation model (Strother, 1986), or the power-law model (Woolley, 2001). The stiffness model and crush of the collision-involved vehicles are then used to calculate the energy absorbed, which is related directly to impact speed and delta-V.

The run-out analysis method uses the documentation of tire-marks and tire-friction against the road, to calculate the post-crash kinetic energies and speeds. The conservation of momentum is then applied to attain the pre-collision velocities, impact speed and delta-Vs of the involved vehicles.

Reconstruction of accidents may be used to relate injury severity to accident severity for statistical or other research purposes. Accident reconstruction may also be applied in product liability or other litigation cases, where the objective is to establish the severity of the accident and the injury mechanisms of the occupants.

In one real-world example of a multiple car collision, a compact car (vehicle 2) was rear-impacted by a pickup truck (vehicle 1) and subsequently pushed into two vehicles (vehicles 3 and 4) in front of it. The damage to the compact car's rear resulted in fatal injuries to the driver. It was unclear whether the rear damage to the compact car was a result of the initial collision with the pick-up truck alone, or if the damage was enhanced by the subsequent collisions.

In another real-world accident, the driver of a large car was severely injured after being sandwiched between a van (on the rear) and a compact car (on the front), which in turn had previously rear-impacted a pick-up truck. The issue was to determine the order in which the driver experienced the collisions and the contribution of each collision to the driver's injury.

Multiple-vehicle accidents involving 3 or more vehicles comprise 13.8% of the severe or fatally injured occupants in fatal accidents (FARS 1998), although single and two-vehicle collisions are the most common type of fatal accidents (93.3% in FARS 1998). The occupants of vehicles in multiple-car collisions are subjected to various injury mechanisms in a complex environment of acceleration pulses and possibly increasing intrusion. Severe injuries may occur in several collision pairs in the car pile-up.

Accident reconstruction of multiple-vehicle accidents is not trivial due to the complexity of the tire-marks on the scene, and the possibility of not having all vehicles available for damage inspection. Crush may result from multiple impacts, which causes difficulty in distinguishing between the damage energy and accident severity of each of the subsequent collisions.

Presently, applications of accident reconstruction techniques for multiple vehicle collisions have not been published, despite the high injury probability in these accidents and the complexity of this type of accidents. Furthermore, no alternatives have been presented to reconstruct multiple-car accidents for which not all vehicles are available for inspection, or where the tire-marks are not well documented.

Possible methods for reconstructing multiple-vehicle collisions need to be available. A technique that reconstructs the severity of a collision pair of interest would be especially fruitful if it allowed the use of the damage energy method applied to only the two vehicles of that pair.

OBJECTIVES

The objective of this paper was to present reconstruction techniques to determine accident severity in multiple car collisions. An analysis was presented to determine conditions under which alternative methods of lower complexity can be applied to reconstruct delta-Vs of specific vehicles of interest, without the need for knowledge of crush and tire-marks of all vehicles.

METHOD

NOTATION

n	Collision possibly causing additional crush to the previously collided vehicle pair
$n-1$	Collision of interest to be reconstructed
$k, k+1$	Vehicles colliding in collision n, with the front of vehicle k into the rear of vehicle k+1
$k-1, k$	Vehicles colliding in collision n-1 with the front of vehicle k-1 and rear of vehicle k, and for which it is determined if it sustained additional crush in collision n as compared to collision n-1
i	Vehicle number variable, with i=1, 2, …, k+1
M_k	Mass of vehicle k
V_1, V_k	Speed of vehicle one, k, respectively
$V_{post, n-1}$	Vehicle speed post impact n-1
$E_{kinetic, pre,n}$	Kinetic energy approaching collision n
$E_{kinetic, post,n}$	Kinetic energy post collision n
$E_{abs, n}$	Energy absorbed in collision n
$K_{k, k+1}$	Combined stiffness of vehicle k's front and vehicle k+1's rear
D_i	Distance vehicle i moves during a collision
$X_{k, k+1\|n}$	Combined or mutual dynamic crush of vehicle k's front and vehicle k+1's rear in crash n
$\Delta X_{k, k+1\|n}$	Mutual additional crush sustained by vehicle k-1's front and vehicle k's rear during crash n
$F_{k,k+1\|n}$	Force experienced by the front of vehicle k and by the rear of vehicle k+1 in collision n
ΔS_i	Braking distance traveled by vehicle i with friction f_i
f_i	Tire to road friction of vehicle i

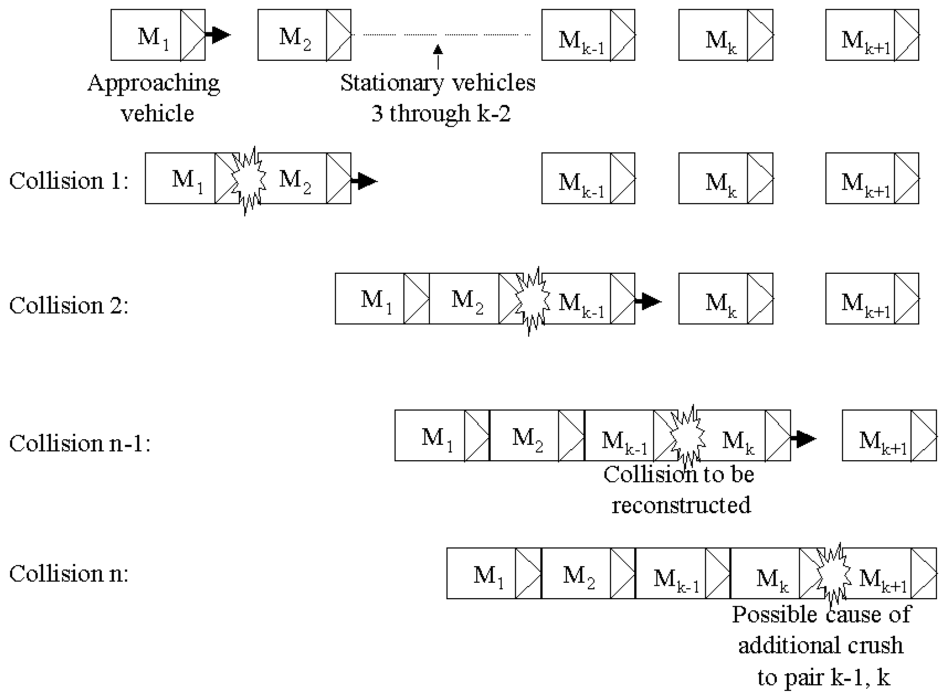


Figure 1. Order of vehicle and collision numbers

ASSUMPTIONS

A multiple-car collision was considered analytically, in which the vehicle pile-up results from a single vehicle approaching a line of stationary, undamaged vehicles. The initial collision between the approaching car and the vehicle rear-most in line causes both vehicles to collide with the vehicle in front of them. This may repeat itself a number of times, depending on the initial speed of the approaching vehicle and the vehicle masses. Figure 1 illustrates the order of events and the corresponding numbering of the vehicles in this type of multiple car collision. The vehicles were modeled as point-masses with mass-less linear springs to model rear and front structures. Restitution was neglected. The distance between the vehicles was assumed to be sufficient for a collision pair to have established full engagement before the successive vehicle is involved in the accident. This is often the case in real-world accidents, since the full engagement is generally established after approximately 0.1 s, during which time vehicles often travel less than 1 to 2 meters.

ACCIDENT RECONSTRUCTION PRINCIPLES

Accident reconstruction is based on laws of physics to establish a relationship between collision severity or vehicle delta-V on the one hand and post-crash tire-marks (momentum method) or vehicle damage (damage energy method) on the other hand.

The momentum method applies the conservation of energy to calculate the vehicles' post impact speeds from post-crash tire-marks, by calculating the friction work of the tires with the road. Successively, the pre-impact speed is calculated from the post-impact speeds and vehicle masses applying the conservation of momentum. The difference in pre and post-impact speed for each vehicle is the accident severity or delta-V experienced by the occupants.

In the application of the damage energy method, first a relationship is found between closing speed (or impact speed) and the energy absorbed by vehicle damage, using the conservation of energy and momentum. Second, crash test data and stiffness models are applied to relate the total energy absorbed by the vehicles involved to the sustained vehicle damage. Third, the two relationships are combined to determine the impact or closing speed from the vehicles' damage. Finally, the delta-V experienced by the vehicles is calculated from the impact speed and the vehicle masses.

In multiple car accidents fulfilling the above conditions, the total accident consists of a series of two-car collisions, where the approaching vehicle may consist of a combination of previously collided vehicles. The damage energy method may be applied directly to reconstruct the collision between a pair of vehicles provided that the damage of this vehicle pair is not enhanced by the successive collision. However, the vehicle pair would sustain additional crush if the impact force on the pair's vehicle structures is greater in the successive crash then in the initial impact.

The three steps of the damage energy method are complemented by an application of Newton's second and third laws to determine the relative impact forces on the vehicle pair's structures in the initial and successive collision.

The next sections expand on the accident reconstruction principles to calculate accident severity of a vehicle pair in a multiple car collision.

KINETIC AND ABSORBED ENERGY VERSUS SPEED

The principles of conservation of momentum and energy are applicable to each collision, and can be used to relate the amount of absorbed or damage energy to vehicle mass and impact speed.

Conservation of momentum states that the pre- collision momentum equals the total momentum post crash (e.g. in collision n):

$$\sum_{i=1}^{k+1} M_i V_{i,pre} = \sum_{i=1}^{k+1} M_i V_{i,post} \qquad \text{(i)}$$

In collision n, the pre-collision kinetic energy equals the summation of the post-crash kinetic energy plus the energy absorbed in collision n, in accordance with the conservation of energy. The post-crash kinetic energy may be partially reduced by the friction work of the tires against the road, before the next collision is entered with pre-collision kinetic energy n+1:

$$E_{kinetic,pre,n} = E_{kinetic,post,n} + E_{absorbed,n} \qquad \text{(iia)}$$

$$E_{kinetic,post,n} - W_{tires,n} = E_{kinetic,pre,n+1} \qquad \text{(iib)}$$

The kinetic energy approaching collision n is equal to the mass of vehicles 1 through k times their approaching or impact velocity, V_i, squared:

$$E_{kinetic,pre,n} = \tfrac{1}{2}\sum_{i=1}^{k} M_i \; V_i^2 \qquad \text{(iii-a)}$$

It was assumed that tire friction energy during each collision can be neglected, as well as the restitution. Negligible restitution causes that the approaching, previously collided vehicles have equal speed. The kinetic energy entering collision n can be written as:

$$E_{kinetic,pre,n} = \tfrac{1}{2}\sum_{i=1}^{k} M_i \;\; V_{pre,n}^2 \qquad \text{(iii-b)}$$

The kinetic energy post-collision n equals:

$$E_{kinetic,post,n} = \tfrac{1}{2}\sum_{i=1}^{k+1} M_i \;\; V_{post,n}^2 \qquad \text{(iii-c)}$$

The damage energy absorbed equals (iii-b) minus (iii-c) when tire forces are neglected. The absorbed energy in a central collision n is then related to half the mass of all vehicles and the impact speed, V_n, squared (using eq. (i)):

$$E_{abs,n} = \frac{1}{2}\,\frac{\sum_{i=1}^{k} M_i \;(M_{k+1})}{\sum_{i=1}^{k+1} M_i}\;\; V_{pre,n}^2 \qquad \text{(iv)}$$

Finally, the friction work of the tires is linearly related to the friction coefficient between tires and road, and the distance traveled at that friction post-collision:

$$W_{tires,n} = \sum_{i=1}^{k} (M_i \;\, f_i)\; g\; \Delta S_n \qquad \text{(v)}$$

ENERGY VERSUS CRUSH

A relationship between the vehicle impact speed and vehicle crush can be attained with equations (i) through (v) and with expressions of energy versus crush. The energy versus crush relationship is based on a force-deflection characteristic model of the vehicles' front and rear structures. The collision force is assumed to be linearly related to the vehicle crush or damage:

$$F_{k,k+1|n} = K_{k,k+1} X_{k,k+1|n} \qquad \text{(vi-a)}$$

The stiffness and deflection in equation (vi) are the combined stiffness and mutual deflection of vehicles k's front and k+1's rear (Figure 2):

$$K_{k,k+1} = \frac{K_k K_{k+1}}{K_k + K_{k+1}} \qquad \text{(vi-b)}$$

$$X_{k,k+1|n} = X_{k|n} + X_{k+1|n} = D_{k+1} - D_k \qquad \text{(vi-c)}$$

The energy absorbed by the vehicle crush in collision n is the integral of force and crush (equation (vi)), when no additional energy is absorbed by the previous collision pair, k-1 and k. This results in:

$$E_{abs,n} = \int F_{k,k+1|n} \; dX_{k,k+1|n} = \tfrac{1}{2} K_{k,k+1} X_{k,k+1|n}^2 \qquad \text{(vii)}$$

Comparison of equation (iv) and (vii) relates the amount of mutual crush in collision pair k, k+1 to the impact speed in that collision, provided that the previous collision pairs do not sustain additional crush in collision n:

$$X_{k,k+1}^2 = \frac{\sum_{i=1}^{k} M_i \ (M_{k+1})}{K_{k,k+1} \ \sum_{i=1}^{k+1} M_i} \quad V_{pre,n}^2$$

(viii)

This is in similarity to the damage energy method applied to a two-car central collision. The absorbed energy needs to be expressed in crush of both vehicle pairs k, k+1 and k-1, k, in case the previous collision pair does experience additional crush in collision n:

$$E_{abs,n} = \tfrac{1}{2} K_{k,k+1} X_{k,k+1|n}^2 + \tfrac{1}{2} K_{k-1,k} X_{k-1,k|n}^2 - \tfrac{1}{2} K_{k-1,k} X_{k-1,k|n-1}^2 \qquad \text{(ix)}$$

The last term on the right-hand side of equation (ix) can not be measured, and the impact speed in collision n can not be determined directly from measured post-accident crush. The occurrence of additional crush requires the force sustained by pair k-1, k in collision n to be higher than that experienced in collision n-1. The force experienced by pair k-1, k in collision n, can be expressed in terms of the force sustained by pair k, k+1 in the same collision with the aid of Newton's second and third laws.

NEWTON'S LAWS

Newton's second and third laws were applied to the line of vehicles to analyze the distribution of forces between the front and rear structures of two successive collision pairs (Figure 2). A linear stiffness model (Campbell 1974) was assumed to model the front and rear stiffness of each of the vehicles (vi).

Newton's second law states that a vehicle's acceleration or delta-V is a result of the sum of the force-vectors acting on that vehicle:

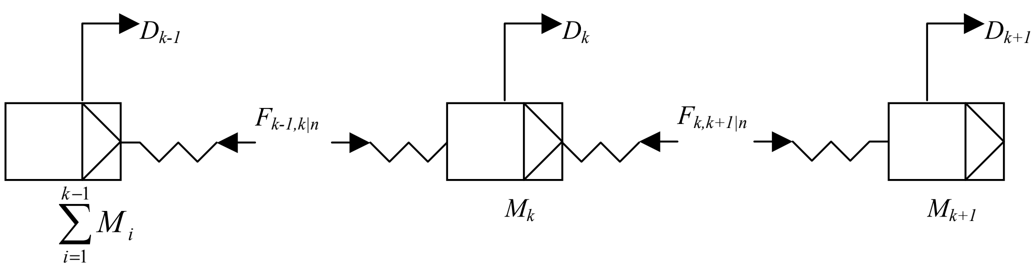


Figure 2. Force balance on vehicles in a multiple vehicle collision

$$\sum_j F_{ij} \ \Delta t = M_i \ \Delta V_k \qquad \text{(x)}$$

Newton's third law states that the force acting on the front structure of vehicle k is equal to the force acting in opposite direction on the rear of vehicle k+1.

Newton's second and third laws were applied to Figure 2:

$$F_{k-1,k|n} \Delta t = -\sum_{i=1}^{k-1} M_i \ \Delta V_{k-1|n} \qquad \text{(xi-a)}$$

$$(F_{k,k+1|n} - F_{k-1,k|n}) \Delta t = \ - M_k \Delta V_{k|n} \qquad \text{(xi-b)}$$

$$F_{k,k+1|n} \Delta t = \ M_{k+1} \Delta V_{k+1|n} \qquad \text{(xi-c)}$$

It was assumed that all previously collided vehicles experienced the same delta-V (ΔV) in collision n, as applied in equation (vi-a). The time duration of the contact forces was assumed equal for all vehicles during the collision. The force experienced by vehicle (k-1)'s front and vehicle k's rear, $F_{k-1,k}$, relates to that experienced by vehicles k and (k+1)'s front and rear , $F_{k,k+1}$. (substitute (xi-a) in (xi-b) and divide result by (xi-a)):

$$\frac{F_{k,k+1|n}}{F_{k-1,k|n}} = \frac{\sum_{i=1}^{k} M_i}{\sum_{i=1}^{k-1} M_i} \qquad \text{(xii)}$$

RESULTS

Equations (i) to (xii) were applied in the reconstruction of multiple car collisions under the set assumptions, for the following conditions:

1. all involved vehicles are available for inspection and have relevant stiffness data available; all tire-marks and the corresponding friction coefficients were well documented.

2. Only the vehicle pair of interest (vehicles k-1 and k) is available for inspection, and relevant stiffness data is available for this pair. The forces in collision n or higher caused no additional crush to this pair.

3. Only the vehicle pair of interest (vehicles k-1 and k) is available for inspection, and relevant stiffness data is available for this pair. The forces in collision n or higher caused additional crush to this pair.

CONDITION 1)

The first method shows that the velocity of the approaching vehicle can be calculated from the summation of all damage energy and friction work, provided that all tire-marks were well documented and all involved vehicles were available for inspection.

The kinetic energy of vehicle 1 can be expressed in terms of the damage energy and tire-work of all vehicles (see Appendix I).

$$E_{kinetic,pre,1} = \tfrac{1}{2} M_1 V_{pre,1}^2 = \sum_{i=1}^{n+1} (E_{abs,i} + W_{tires,i})_{post} \quad \text{(I)}$$

The absorbed energy, $E_{abs,i}$, can be written in terms of each vehicle pair's front and rear crush (II), and the friction energy can be written as the friction work of each vehicle's tires with the road (III) :

$$E_{absorbed,i} = (\tfrac{1}{2} K_{i,front} X_{i,front}^2 + \tfrac{1}{2} K_{i+1,rear} X_{i,rear}^2) \quad \text{(II)}$$

$$W_{tires,i} = 2\ f_i\ g\ \Delta S_i \quad \text{(III)}$$

The accident severity and delta-V can be calculated for each set of vehicles in line (collision pair), once the kinetic energy of the approaching vehicle has been determined:

$$V_{imp,j+1} = \sqrt{2E_{kinetic,pre,j+1} / \sum_{i=1}^{j} M_i} \quad \text{(IV)}$$

,with j=0,..,n-1, and:

$$E_{kinetic,pre,j+1} = E_{kinetic,pre,j} - E_{absorbed,j} - W_{tires,j}$$

$$\Delta V_{k|n} = M_{k+1}\ V_{imp,n} / \sum_{i=1}^{k} M_i \quad \text{(V)}$$

$$\Delta V_{k+1|n} = \sum_{i=1}^{k} M_i\ V_{imp,n} / \sum_{i=1}^{k+1} M_i \quad \text{(VI)}$$

The kinetic energy of the approaching vehicle can be calculated from the damage or crush energy absorbed by each vehicle's front and rear crush, plus the run-out work from the tire-to-road friction. The delta-V for each vehicle can be calculated in each impact, and the complete set of injury mechanisms resulting from the multiple set of impacts can be examined. This method requires inspection of each vehicle's front and rear damage and the availability of the relevant stiffness data for each vehicle. Furthermore, this calculation needs knowledge of the braking force applied by each vehicle's driver, the consequent friction with the road, and the braking or skid distance of each of the vehicles involved.

CONDITION 2)

The data required for the above-presented method are not always available.

A more efficient way to reconstruct the initial accident severity and delta-V of a collision pair would be to apply the damage energy method to just the two vehicles of interest. Such a method would be particularly fruitful when the initial frontal or rear impact sustained by the vehicle of interest is believed to be the primary cause of the occupant injuries. This method requires the conditions that a) the multiple-vehicle collision can be considered as a sequence of separate two-vehicle collisions, and b) that the damage to the collision pair was a result from solely the first impact (impact n-1) between the subject pair (consisting of vehicles k-1 and k).

The collision pair would not experience additional crush in a collision (n) following the initial impact (n-1) in case the impact force in crash n would not exceed that of crash n-1. The impact forces on the collision pair of interest are limited by the energy to be absorbed in the impact and are further determined by the relative stiffness of the vehicles absorbing that energy and the relative forces acting on each vehicle's structures.

The derivations in Appendix II show that the original front and rear crush of vehicles k-1 and k in collision n-1 is not enhanced by the subsequent collision n if:

$$\left[\frac{\sum_{i=1}^{k-1} (M_i)\ M_{k+1}}{\sum_{i=1}^{k+1} (M_i)\ M_k} \right] < \left[\frac{\sum_{i=1}^{k} (M_i)}{\sum_{i=1}^{k-1} (M_i)} \right]^2 \frac{K_{k-1,k}}{K_{k,k+1}} \quad \text{(VII)}$$

It can be proven that each impact subsequent to collision n will less likely cause additional crush to the subject collision pair k-1, k. This proof can be attained by increasing the value of n and k to the new impact-number (n+j), and by substitution of k-j for the new k in all terms except for the mass denominator term on the right hand.

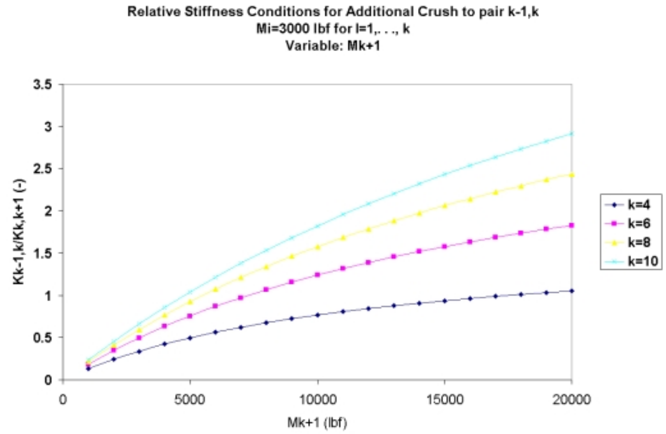
All front and rear damage to the vehicles of interest (vehicles k-1 and k, respectively) resulted from the initial collision between those two vehicles (collision n-1), provided that the assumptions previously stated and condition (VII) apply. The impact speed and velocity changes experienced by the subject vehicle pair can then be calculated applying the damage energy method directly to those two vehicles (all prior vehicle masses must be known):

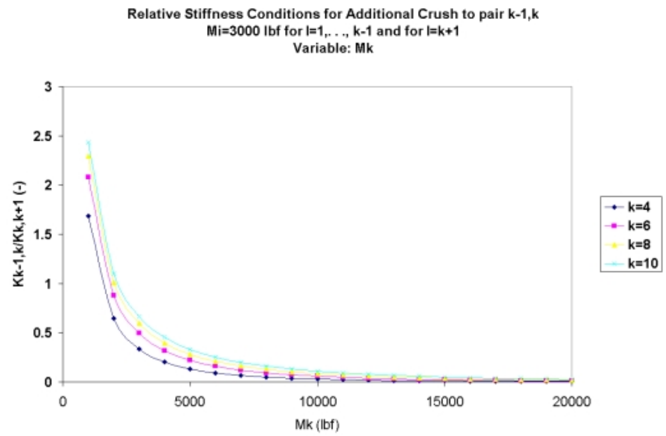
$$E_{abs,k-1,k} = \frac{1}{2}\,\frac{\sum_{i=1}^{k-1} M_i \,(M_k)}{\sum_{i=1}^{k} M_i}\quad V_{k-1,k}^{2} \qquad \text{(VIII)}$$

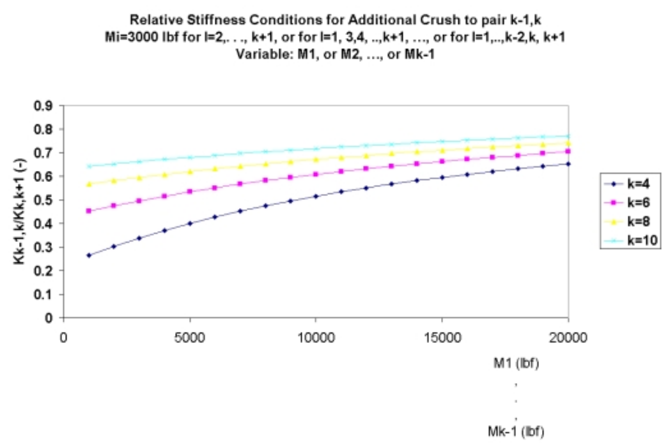
$$\begin{cases} \Delta V_{k|n-1} = \dfrac{\sum_{i=1}^{k-1} M_i}{\sum_{i=1}^{k} M_i} V_{k-1,k} \\ \Delta V_{k-1|n-1} = \dfrac{M_k}{\sum_{i=1}^{k} M_i} V_{k-1,k} \end{cases} \qquad \text{(IX)}$$

The condition for no additional crush to collision pair k-1, k in collision n and successive impacts is generally fulfilled for a multiple car pile-up consisting of vehicles with masses ranging anywhere from small passenger cars to heavy tractor-trailers. However, it is required that the vehicles have reasonable real-world stiffness characteristics and that the mass of the vehicle to be impacted is of similar magnitude as that of the prior vehicles.

The mass-effect of some of the vehicles is further explored in figures 3 to 5 for multiple-car collisions with various numbers of vehicles involved. Figure 3 shows the minimum stiffness ratio of vehicle pair k-1,k versus pair k, k+1 to ensure that no additional crush is sustained by pair k-1,k in collision n, for varying vehicle mass k+1, in case all other vehicles are of equal mass (e.g. 3000 lbf). No additional crush is sustained by the vehicles of interest in collision n, when the actual stiffness ratio of vehicle pair k-1,k versus that of pair k, k+1 is above the line that corresponds to the number of impacts in the multiple car collision. The effect of vehicle masses Mk, or Mi for i< 1, 2,…, k-1 are presented in similar graphs (Figures 4 and 5, respectively).







Figures 3, 4 and 5 show the minimum stiffness ratio of pair k-1,k versus pair k, k+1 for a varying vehicle mass, M_{k+1} (Figure 3), M_k (Figure 4), or one of the other vehicles (Figure 5), to ensure that the total crush of pair k-1, k is the sole result from collision n-1. It is assumed that all other vehicles have equal mass (3000 lbf in figures 3-5). Each line represents the number of impacts in the multiple-car collision. For example, consider a 9-car collision (8 impacts), with cars 1 through 8 having a mass of 3000 lbf, and vehicle 9 having a mass of 15,000 lbf. The front crush of vehicle 7 and the rear crush of vehicle 8 resulting from impact 7 is not enhanced in impact 8, when the ratio of the combined front and rear stiffness of vehicles 7-8 and of vehicles 8 –9 is greater than 2.06.

CONDITION 3)

The previous method can not be applied to reconstruct the accident severity of pair k-1 and k if e.g. k passenger cars approach a tractor-trailer (vehicle k+1), and the stiffness of pair k-1,k is not sufficient to prevent additional crush in impact n (see Figure 3). The initial impact speed and delta-Vs in collision n-1 would be overestimated if the damage energy method were directly applied to the crush of pair k-1, k, since the post-accident crush is a result of both collisions n-1 and n.

The derivation in Appendix III shows that the impact speed in collision n-1 can be expressed in terms of the post-crash crush of vehicle pairs k-1,k and k,k+1, plus the tire-work from all vehicles between points of impact n-1 and n:

$$V_{pre,n-1}^{2} = \left(\frac{\sum_{i=1}^{k} M_i \sum_{i=1}^{k+1} M_i}{\sum_{i=1}^{k-1} M_i \left(\sum_{i=1}^{k+1} M_i M_k + \sum_{i=1}^{k-1} M_i M_{k+1} \right)} \right) * \left(K_{k,k+1} X_{k,k+1|n}^{2} + K_{k-1,k} X_{k-1,k|n}^{2} + \frac{M_{k+1}}{\sum_{i=1}^{k+1} M_i} W_{tires,n} \right) \quad \text{(X)}$$

DISCUSSION

The reconstruction of multiple vehicle accidents is a complex task, since many vehicles are involved. Residual crush may be the result of a combination of subsequent collisions. Occupants of several collision pairs may have been severely injured, and injury mechanisms may have been complicated by the sustained series of rear and front impacts. This study examined various methods to reconstruct multiple vehicle accidents, resulting from one vehicle approaching a line of stationary undamaged vehicles.

The first method allows the reconstruction of the impact speeds and delta-Vs of the complete line of vehicles, by calculating the damage energy plus the run-out energy of all vehicles. It is an effective tool, especially when the delta-Vs and occupant injury mechanisms of several collision pairs need to be determined. The method requires that all vehicles are available for inspection of the damage to front and rear, and that the corresponding relevant stiffness data can be acquired. Furthermore, knowledge of each driver's braking force and consequent friction with the road is needed, as well as the braking distance. The accuracy of the reconstruction is limited to the accuracy of the damage and tire-mark measurements, as well as stiffness data and friction coefficient estimates. However, the accuracy is greatly reduced when not all vehicles are available for inspection. In these cases, crush of some vehicles has to be estimated from photographs. Other vehicles' crush may be determined from the collision partner's crush, and the stiffness ratio of the collision pair, using Newton's third law (Hull 1993, Neptune and Flynn 1994, Grimes et al. 1997 and Long 1999). The accuracy may be further reduced when the run-out distances are not well documented, or when some vehicles did not have locked tires and did not leave skid-marks. The run-out energy may have to be guessed from estimated braking distances and forces, which may introduce great errors. This method may not be sufficiently accurate and effective to reconstruct velocity changes and occupant injury mechanisms of one particular collision pair.

The second method is very effective when the initial front or rear-impact of a vehicle in a multiple-car collision is believed to be the primary cause of the occupant injuries. The main objective of the accident reconstruction is to determine the delta-V of this initial impact. The method presented the conditions under which the accident severity, delta-Vs and occupant injury mechanisms of the initial and most severe rear impact can be determined from the damage energy analysis applied to the two vehicles of interest only.

The conditions require a minimum stiffness for the collision pair of interest compared to that of the next pair (ahead), while the stiffness requirement depends on all vehicle masses.

Figure 6. A real-world example of a multiple-car collision resulting from a vehicle approaching a line of stationary undamaged cars

Figure 6 shows a picture of a real-world multiple-car collision. In this collision, a 1995 Dodge Dakota approached a line of stationary vehicles. The Dakota did not manage to stop in time and caused a multiple car collision in which 3 other vehicles were involved. The driver of vehicle 2 (see figure 1) received injuries that were a result of the initial rear impact by the Dakota. The objective of the accident reconstruction was to determine the accident severity of the initial impact between the Dodge and the Chevette and the delta-V of the Chevette in that collision. The Dakota and Chevette were the only two vehicles available for inspection.

Table 1: Vehicle numbers, masses and stiffnesses for a real-world accident multiple car collision

#	Model	Mass (lbs)	Front Stiffness (lbf/inch^2)	Rear Stiffness (lbf/inch^2)
1	Dodge Dakota	4,000	73.3	NA
2	Chevrolet Chevette	2,400	63.0	61.5
3	Buick LeSabre	3,700	Unknown	70.0
4	Honda Prelude	2,500	Unknown	Unknown

Condition (VII) was applied to determine whether the Dakota front and Chevette rear structure sustained additional crush in the successive impact between the Chevette front and the LeSabre rear. The reconstruction of the first collision between vehicles 1 and 2 is requested, such that k-1=1, and thus k=n=2. We will therefore only consider the collisions between the first 3 vehicles, although the complete accident was a 4-car crash. Application of condition (VII), with n=k=2, states that vehicles 1 and 2 would sustain no additional crush in the impact with vehicle 3 for the following stiffness requirement (the vehicle masses and stiffnesses are found in Table 1):

$$\frac{M_1 M_3}{(M_1 + M_2 + M_3) M_2} < \left(\frac{M_1 + M_2}{M_1} \right)^2 \frac{K_{1,2}}{K_{2,3}}$$

(substitute values from Table 1)

$K_{1,2} / K_{2,3} > 0.61056 / 2.56 = 0.2385$

Comparison of the true $K_{1,2}/K_{2,3}$ with the requirement indicates that the condition was fulfilled and that no additional crush was caused to vehicle one's front and vehicle two's rear, by colliding with vehicle 3. It was assumed that the subsequent collision with vehicle 4 would not contribute to the crush of vehicles 1 and 2, as under most situations a collision n+j (with j>0) will less likely contribute to the damage of vehicle pair k-1, k than collision n. This can be proven by increasing the value of n and k to the new impact-number (n+j), and by substitution of k-j for the new k in all terms of eq. (VII) except for the mass denominator term on the right hand. The proof shows that the stiffness requirement of equation (VII) is more likely fulfilled for collisions subsequent to n. The accident reconstruction of the collision between the Dakota and the Chevette could be completed by direct application of the damage energy method to determine the impact speed of the Dakota and the delta-V of the Chevette, using equations (VIII) and (IX).

The effect of the Dakota mass is examined in Figure 7. The line represents the minimum stiffness ratio for the Dakota front and Chevette rear compared to that of the Chevette front and LeSabre rear, to predict no additional crush for vehicles 1 and 2 in the second impact. Figure 7 shows that no additional crush is expected if the actual stiffness ratio of the collision pairs is above that line. Application of the stiffness data in Table 1 shows that the stiffness ratio for this real-world accident is well above the line (see enlarged data-point in Figure 7).

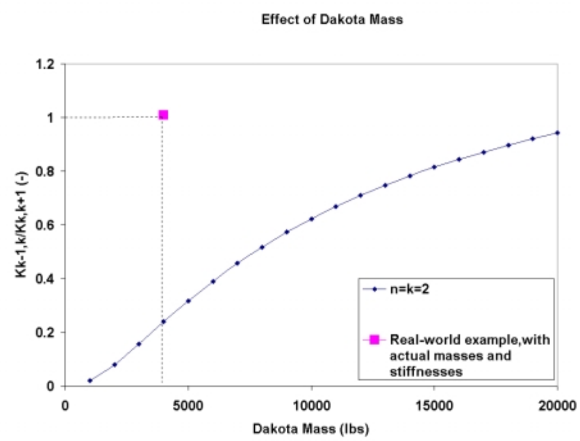


Figure 7: Effect of the Dakota or vehicle 1 mass on the minimum stiffness for the real-world example.

It should be noted, that the method was developed for severe collisions in which the restitution can be neglected. The method is not applicable for reconstruction of low-speed multiple car accidents, where restitution often plays an important role.

The analysis under condition 2) can also be very useful in another set of multiple car collisions, in which a vehicle approaches a line of stationary, previously accident-involved vehicles (Figure 8). This type of multiple car collision may typically occur when a vehicle is stopped in a fog-bank, and the approaching vehicles one by one cause a collision into the stationary, (previously collided set of) vehicles.

Assume that the vehicle of interest (vehicle k) collided with its front to the rear of a previously collided, stationary vehicle (k-1). It is again assumed that restitution is negligible, such that the previously collided vehicles are still connected post-crash. Successively, another vehicle (k+1) approaches the rear of vehicle k. The resulting collision (n) may enhance the frontal crush the vehicles of interest received previously, provided that the collision force exceeds that of the previous crash. The force of the rear-most pair is greater than that in front of it (with application of eq. (xii)):

$$\frac{F_{k,k+1|n}}{F_{k-1,k|n}}=\frac{\sum_{i=1}^{k}M_i}{\sum_{i=1}^{k-1}M_i} \qquad \text{(XI)}$$

Application of the linear stiffness model gives an expression of the crush ratio of pair k, k+1 and k-1,k that would result from the impact between the front of vehicle k+1 and the rear of vehicle k, in case pair k-1 and k had not sustained crush previously:

$$X_{k,k+1|n}=\frac{\sum_{i=1}^{k}M_i}{\sum_{i-1}^{k-1}M_i}\frac{K_{k-1,k}}{K_{k,k+1}}X_{k-1,k|n} \qquad \text{(XII)}$$

Equation (XII) shows that the forces of the last collision (n) would cause a greater damage to pair k,k+1 than to pair k-1,k, provided that stiffness $K_{k-1,k}$ is similar to or greater than $K_{k,k+1}$. However, collision n-1 may already have resulted in damage to pair k-1, k greater than that predicted for collision n (equation (XII)). In that case, the impact force in collision n was not sufficient to increase the post-crash damage in k-1,k from collision n-1 and the damage in pair k-1, k must have resulted solely from collision n-1, provided that the stiffness of both pairs is similar. The kinetic energy change involved in collision n was then absorbed by the crush of vehicles k and k+1 alone. The impact speeds of vehicle k in collision n-1 and of vehicle k+1 in collision n can be determined independently using equation (VIII). On the other hand, pair k-1, k has most likely sustained additional crush in impact n if this pair's measured crush corresponds to that of pair k, k+1 in accordance with equation (XII). The kinetic energy change in collision n was then absorbed by the crush of pair k, k+1 and by the additional crush of pair k-1, k.

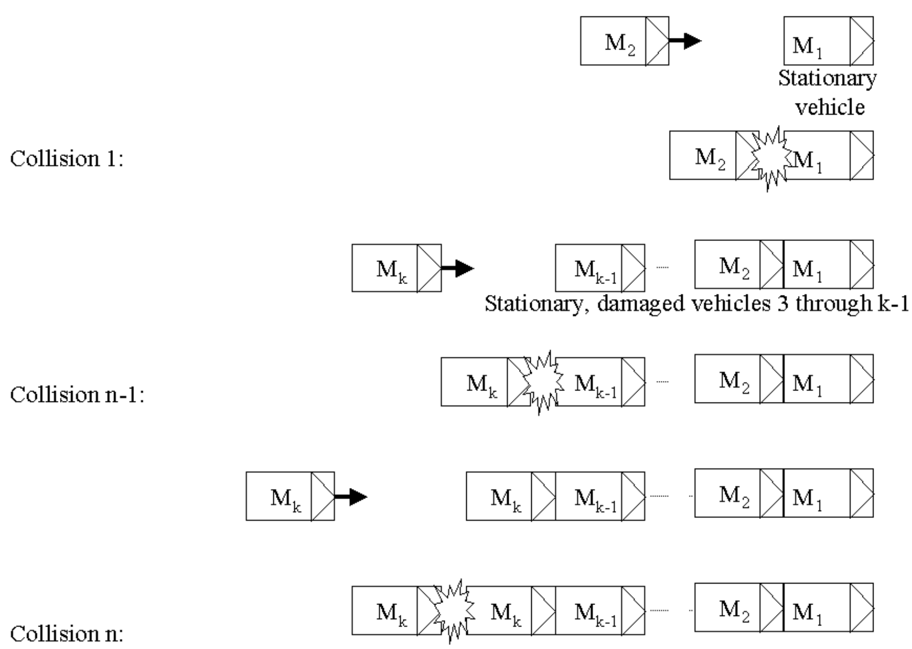


Figure 8. Order of vehicles in a multiple car collision resulting from a vehicle approaching a line of previously accident-involved vehicles

Figure 9 shows an example of a real-world multiple-car collision of this type. A Honda Accord station wagon came to (almost) a stop on the freeway, and was rear-impacted by a Plymouth Voyager. Successively, an oncoming Toyota Starlet rear-impacted the Voyager and finally, the Toyota was rear-impacted by an approaching Pontiac Grand-Prix. Each rear-impact occurred after the previously collided vehicles had come to a stop. The objective of the accident reconstruction was to determine the impact speed of the Toyota Starlet. The photos and vehicle measurements revealed that the crush of the Pontiac's front and Toyota's rear structure was significantly less than that of the Toyota's front and Dodge's rear. This indicated that the forces of the rearmost collision were insufficient to cause additional crush to the Toyota front and Dodge rear, and the impact speed of the Toyota could be determined from application of the damage energy method to the damage of the Toyota front and Dodge rear.

Figure 9. A real-world example of a multiple car collision resulting from a vehicle approaching a line of previously accident-involved vehicles.

The third method showed the possibility of reconstructing the accident severity and delta-V for one vehicle pair (k-1,k) based on measurements of only that pair's crush, the crush of pair (k, k+1) and the tiremarks between collision n-1 and n. This method requires the inspection of three vehicles, as well as stiffness data for the front structure of all three and rear structure of vehicle k. Furthermore, assumptions have to be made for the friction coefficient of the tires with the road between points of impact n-1 and n. This is still a more accurate and effective method than the first method, especially is the collision involved many vehicles.

CONCLUSIONS

The methods presented in this paper drastically simplify and improve the accident reconstruction of vehicle pairs in certain types of multiple-car-collisions.

A complete multiple car collision resulting from one car approaching a line of undamaged stationary vehicles can be reconstructed accurately, when all vehicles are available for inspection, when all relevant stiffness data is available, and when all tire-marks and corresponding friction coefficients are well known.

In this type of multiple car collisions, more effective methods can be applied to reconstruct the accident severity for one vehicle pair of interest and for which it is believed that the initial rear-impact caused the main occupant-injuries.

The damage energy method, as generally applied in two-car collisions, can be applied directly to only the vehicle pair of interest under certain stiffness and mass conditions. The conditions apply for most real-world situations. However, the conditions may not be fulfilled when one or more of the vehicles is significantly heavier and/or stiffer than the vehicles of interest. Furthermore, the method's application is restricted to severe collisions in which the restitution can be neglected.

The accident severity and delta-V of a vehicle pair can be reconstructed accurately from only the crush and relevant stiffness data of the three vehicles in vehicle pairs k, k+1 and k-1,k, when the above requirements are not met.

A multiple-car collision resulting from a vehicle approaching a line of stationary, previously collided vehicles may also be reconstructed using the damage energy method directly to only the vehicles of interest. This may be done for cases in which the crush of the previously collided vehicles is greater than that of the newly impacted vehicle pair (assumed that the vehicle pairs have similar stiffness).

ACKNOWLEDGMENTS

The author would like to thank Ronald L. Woolley for his fruitful suggestions and moral support

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APPENDIX I

The principles of conservation of momentum and energy are applied to reconstruct a multiple-vehicle collision, where one vehicle approaches a line of stationary vehicles. The kinetic energy is partially absorbed in the collision, and the post-crash kinetic energy is partially absorbed by friction work (see (I), (iia) and (iib)):

$$E_{kinetic,pre,i} = E_{absorbed,i} + E_{kinetic,post,i} \quad \text{(I)}$$

$$E_{kinetic,post,i} - W_{tires,i} = E_{kinetic,pre,i+1} \quad \text{(II)}$$

The kinetic energy of vehicle one can be expressed using the substitution of (II) in (I) repeatedly applying i =1 and the last collision being collision n, between vehicles k and k+1:

$$\begin{aligned} E_{kinetic,pre,1} &= \tfrac{1}{2} M_1 V_{pre,1}^2 \\ &= \sum_{i=1}^{n+1} (E_{absorbed,i} + W_{tires,i})_{post} \end{aligned} \quad \text{(III)}$$

The absorbed energy, $E_{abs,i}$, can be written in terms of each vehicle pair's front and rear crush (IV), and the friction energy can be written as the friction work of each vehicle's tires with the road (V) :

$$E_{absorbed,i} = (\tfrac{1}{2} K_{i,front} X_{i,front}^2 + \tfrac{1}{2} K_{i+1,rear} X_{i,rear}^2) \quad \text{(IV)}$$

$$W_{tires,i} = 2\ f_i\ g\ \Delta S_i \quad \text{(V)}$$

The accident severity and delta-V can be calculated for each set of vehicles in line (collision pair), once the kinetic energy of the approaching vehicle has been determined (using eq. (II)):

$$E_{kinetic,pre,j+1} = \tfrac{1}{2} \sum_{i=1}^{j} M_i\ V_{imp,j+1}^2 \quad \text{(VI)}$$

, with $E_{kinetic,pre,i+1} = E_{kinetic,post,i} - W_{tires,i}$

The impact speed is:

$$V_{imp,j+1} = \sqrt{2 E_{kinetic,pre,j+1} / \sum_{i=1}^{j} M_i} \quad \text{(VII)}$$

$$\Delta V_{k|n} = M_{k+1}\ V_{imp,n} / \sum_{i=1}^{k} M_i \quad \text{(VIII)}$$

$$\Delta V_{k+1|n} = \sum_{i=1}^{k} M_i\ V_{imp,n} / \sum_{i=1}^{k+1} M_i \quad \text{(IX)}$$

The kinetic energy of the approaching vehicle can be calculated from the damage or crush energy absorbed by each vehicle's front and rear, plus the run-out work from the tire-to-road friction. The delta-V for each vehicle can be calculated in each impact, and the complete set of injury mechanisms resulting from the multiple set of impacts can be examined.

APPENDIX II

The next sections present the relationship between the total damage energy to be absorbed in collision n and that in collision n-1, the relative vehicle stiffnesses and the relative force distribution among the line of vehicles.

ENERGY ABSORBED IN COLLISION N VERSUS COLLISION N-1

The energy absorbed in collision n is expressed in the speed of the approaching vehicle(s) (see equation (iv)):

$$E_{abs,n} = \frac{1}{2} \frac{\sum_{i=1}^{k} M_i\ (M_{k+1})}{\sum_{i=1}^{k+1} M_i} \quad V_{pre,n}^2 \quad \text{(X)}$$

The vehicle speed approaching collision n, $V_{pre,n}$, is equal to or smaller than the speed post-impact n-1, according to equation (ii-b), using (equation (iii-a):

$$V_{pre,n} = \sqrt{V_{post,n-}^2 - \frac{W_{tires,n}}{\sum_{i=1}^{k} M_i}} \le V_{post,n-1} \quad \text{(XI)}$$

The upper limit of the energy absorbed in collision n can be expressed in terms of the vehicle speed pre-collision n-1, applying the conservation of momentum of collision n-1:

$$\sum_{i=1}^{k-1} M_i V_{pre,n-1} = \sum_{i=1}^{k} M_i V_{post,n-1} \quad \text{(XII)}$$

$$E_{abs,n} = \frac{\sum_{i=1}^{k} M_i (M_{k+1})}{2\sum_{i=1}^{k+1} M_i} \left\{ \left(\frac{\sum_{i=1}^{k-1} M_i}{\sum_{i=1}^{k} M_i} \right)^2 V_{pre,n-1}^2 - \frac{W_{tires,n}}{\sum_{i=1}^{k} M_i} \right\}, or$$

$$E_{abs,n} \le \frac{\sum_{i=1}^{k} M_i (M_{k+1})}{2\sum_{i=1}^{k+1} M_i} \left(\frac{\sum_{i=1}^{k-1} M_i}{\sum_{i=1}^{k} M_i} \right)^2 V_{pre,n-1}^2 \qquad \text{(XIIIa,b)}$$

Finally, the energy absorbed in collision n can be expressed in that of collision n-1, using equation (X) (with substitution of k-1 for k):

$$E_{abs,n} = \frac{\sum_{i=1}^{k-1} M_i (M_{k+1})}{\sum_{i=1}^{k+1} M_i (M_k)} E_{abs,n-1} - \frac{M_{k+1}}{2\sum_{i=1}^{k+1} M_i} W_{tires,n} \qquad \text{(XIVa,b)}$$

$$E_{abs,n} \le \frac{\sum_{i=1}^{k-1} M_i (M_{k+1})}{\sum_{i=1}^{k+1} M_i (M_k)} E_{abs,n-1}$$

RELATIVE STIFFNESS OR ABSORBED ENERGY

The damage or absorbed energy can be expressed in terms of crush. It is assumed that in collision n-1, the front of vehicle k-2 and the rear of vehicle k-1 did not absorb additional crush, as compared to the crush after collision n-2. It will be shown in this analysis that this assumption is applicable in most real-world situations. Vehicle (k-1)'s front and vehicle k's rear structure sustained a combined crush in collision n-1, Xk-1,k|n-1, and absorbed damage energy Eabs,n-1:

$$E_{abs,n-1} = \tfrac{1}{2} K_{k-1,k} X_{k-1,k|n-1}^2 \qquad \text{(XV)}$$

In collision n, the energy absorbed can be written in terms of crush as well:

$$E_{abs,n} = \begin{cases} = \tfrac{1}{2} K_{k,k+1} X_{k,k+1|n}^2 + \tfrac{1}{2} K_{k-1,k} X_{k-1,k|n}^2 - \\ \tfrac{1}{2} K_{k-1,k} X_{k-1,k|n-1}^2, \ \forall X_{k-1,k|n} > X_{k-1,k|n-1} \\ = \tfrac{1}{2} K_{k,k+1} X_{k,k+1|n}^2 \quad \forall X_{k-1,k|n} = X_{k-1,k|n-1} \end{cases} \qquad \text{(XVIa,b)}$$

The distribution of the absorbed energy in the three right hand terms of (XVIa) can be calculated using Newton's second and third laws.

RELATIVE FORCES

Figure 2 and equation (ix) show the relationship between the forces experienced by the front and rear of vehicles n-1 and n versus those sustained by the front and rear structures of vehicles n and n+1:

$$\frac{F_{k,k+1|n}}{F_{k-1,k|n}} = \frac{\sum_{i=1}^{k} M_i}{\sum_{i=1}^{k-1} M_i} \qquad \text{(XVII)}$$

The forces between vehicles k and k+1, as well as between vehicles k-1 and k can be expressed in terms of crush, based on a linear relationship between force and total mutual crush (see equation (ix-a)):

$$\begin{aligned} F_{k-1,k|n} &= K_{k-1,k} X_{k-1,k|n} \\ &= K_{k-1,k} (X_{k-1,k|n-1} + \Delta X_{k-1,k|n}) \end{aligned} \qquad \text{(XVIIIa)}$$

$$F_{k,k+1|n} = K_{k,k+1} X_{k,k+1|n} \qquad \text{(XVIIIb)}$$

The total crush of pair k-1, k in collision n can be expressed in that of collision pair k, k+1, using equations (XVII) and (XVIIIa,b):

$$X_{k-1,k|n} = \frac{\sum_{i=1}^{k-1} M_i}{\sum_{i-1}^{k} M_i} \frac{K_{k,k+1}}{K_{k-1,k}} X_{k,k+1|n} \qquad \text{(XIX)}$$

$\Delta X_{k-1,k|n}$ is the additional crush to collision pair k-1 and k, caused in collision n as compared to the crush caused in collision n-1:

$$\Delta X_{k-1,k|n} = \frac{\sum_{i=1}^{k-1} M_i}{\sum_{i=1}^{k} M_i} \left(\frac{K_{k,k+1}}{K_{k-1,k}} X_{k,k+1|n} \right) - X_{k-1,k|n-1}$$

$$\forall \ \Delta X_{k-1,k|n} \ge 0 \qquad \text{(XX)}$$

Equation (XVIa) with substitution of (XV) and (XX) expresses the energy absorbed in collision n in terms of the new collision pair's '(vehicles k, k+1) crush only: results in:

$$E_{abs,n} = \tfrac{1}{2} K_{k,k+1} X^2_{k,k+1|n} (1 + \left(\frac{\sum_{i=1}^{k-1} M_i}{\sum_{i=1}^{k} M_i} \right)^2 \frac{K_{k,k+1}}{K_{k-1,k}}) - E_{abs,n-1}$$

(XXI),

This expression applies under the condition that collision n enhances the crush sustained in impact n-1 by collision pair k-1, k:

$$X_{k-1,k|n-1} < \frac{\sum_{i=1}^{k-1} (M_i)}{\sum_{i=1}^{k} (M_i)} \frac{K_{k,k+1}}{K_{k-1,k}} X_{k,k+1|n} \qquad \text{(XXII)}$$

CONDITION FOR ADDITIONAL CRUSH

Due to the conservation of energy and momentum, the energy absorbed in collision n is equal to or smaller than a portion of that of the previous collision, as given by equation (XIV). Substitution of (XXI) and (XV) in (XIV) results after reorganizing in:

$$\frac{K_{k,k+1}}{K^2_{k-1,k}} (1 + \left(\frac{\sum_{i=1}^{k-1} (M_i)}{\sum_{i=1}^{k} (M_i)} \right)^2 \frac{K_{k,k+1}}{K_{k-1,k}}) \frac{1}{2} K_{k,k+1} X^2_{k,k+1|n}$$
$$\leq \left[\frac{\sum_{i=1}^{k-1} (M_i)\, M_{k+1}}{\sum_{i=1}^{k+1} (M_i)\, M_k} + 1 \right] \frac{1}{2} K_{k-1,k} X^2_{k-1,k|n-1} \qquad \text{(XXIII)}$$

Equation (XXIII) applies only under the condition that collision n causes additional crush to the front of vehicle k-1 and the rear of vehicle k, as given by (XXI). Substitution of (XXII) in (XXIII) gives the final condition for the occurrence of additional crush to vehicles k-1 and k, in subsequent collision n.

Elimination of the crush terms and simplification of (XXIII) shows that the original front and rear crush of vehicles k-1 and k in collision n-1 is not enhanced by the subsequent collision n if:

$$\left[\frac{\sum_{i=1}^{k-1} (M_i)\, M_{k+1}}{\sum_{i=1}^{k+1} (M_i)\, M_k} \right] < \left[\frac{\sum_{i=1}^{k} (M_i)}{\sum_{i=1}^{k-1} (M_i)} \right]^2 \frac{K_{k-1,k}}{K_{k,k+1}} \qquad \text{(XXIV)}$$

It can be proven that each impact succeeding collision n will less likely cause additional crush to the subject collision pair, by increasing the value of n and k to the new impact-number (n+j), and by substitution of k-j for the new k in all terms except for the mass denominator term on the right hand.

All front and rear damage to the vehicles of interest (vehicles k-1 and k, respectively) resulted from the initial collision between those two vehicles (collision n-1), provided that the assumptions previously stated and condition (XXIV) apply. The impact speed and velocity changes experienced by the subject vehicle pair can be calculated applying the damage energy method directly to those two vehicles (all prior vehicle masses must be known):

$$E_{abs,k-1,k} = E_{abs,n-1} = \frac{1}{2} \frac{\sum_{i=1}^{k-1} M_i\ (M_k)}{\sum_{i=1}^{k} M_i} \quad V^2_{pre,n-1} \qquad \text{(XXV)}$$

$$\Delta V_{k|n-1} = \frac{\sum_{i=1}^{k-1} M_i}{\sum_{i=1}^{k} M_i} V_{pre,n-1}$$
$$\Delta V_{k-1|n-1} = \frac{M_k}{\sum_{i=1}^{k} M_i} V_{pre,n-1} \qquad \text{(XXVIa,b)}$$

APPENDIX III

The following derivation shows the possibility of differentiating between the energy absorbed in collision n-1 and that in collision n, using the post-accident crush.

Equation (XVIa) expresses the energy absorbed in collision n in terms of the energy absorbed in vehicles k and k+1, plus the additional crush to vehicles k-1 and k:

$$E_{abs,n} = \begin{cases} = \frac{1}{2} K_{k-1,k} (X^2_{k-1,k|n} - X^2_{k-1,k|n-1}) + \\ \frac{1}{2} K_{k,k+1} X^2_{k,k+1|n}, \forall X_{k-1,k|n} > X_{k-1,k|n-1} \quad (XVIa) \\ = \frac{1}{2} K_{k,k+1} X^2_{k,k+1|n}, \forall X_{k-1,k|n} = X_{k-1,k|n-1} \quad (XVIb) \end{cases}$$

The first two parts of the right-hand term represent the total crush of collision pair k-1 and k, plus that of collision pair k and k+1, after collision n. This crush of both pairs can be measured post-accident. The third part in the right-hand term is equal to the energy absorbed in collision n-1, $E_{abs,n-1}$(Equation (XV)), which is unknown. The left-hand term can be expressed in terms of the energy absorbed in collision n-1 and the tire-work post-crash n-1 (equation (XIV)), such that only one unknown, $E_{abs,n-1}$, can be expressed in terms of vehicle damage of two collision pairs and in terms of tire-work between collisions n-1 and n.

$$E_{abs,n} + E_{abs,n-1} = \frac{1}{2} K_{k,k+1} X^2_{k,k+1|n} + \frac{1}{2} K_{k-1,k} X^2_{k-1,k|n} \quad \text{(XXVII)}$$

Substitution of (XIV):

$$E_{abs,n-1}\left(\frac{\sum_{i=1}^{k-1} M_i \ (M_{k+1})}{\sum_{i=1}^{k+1} M_i \ (M_k)} + 1\right) - \frac{M_{k+1}}{2\sum_{i=1}^{k+1} M_i} W_{tires,n} = \frac{1}{2} K_{k,k+1} X^2_{k,k+1|n} + \frac{1}{2} K_{k-1,k} X^2_{k-1,k|n} \quad \text{(XXVIII)}$$

$W_{tires,n}$ is the tire work against the road of vehicles M_1 through M_k from point of impact n-1 to point of impact n.

Neglecting the tire work gives an upper limit estimate for the impact speed in crash n-1 (see equation XXV).

However, the tire-work between impacts n-1 and n can often not be neglected. In that case, the tire-work between these two impacts needs to be calculated from documented or measured tire-marks on the road and estimated friction coefficient. The initial delta-Vs of pair k-1 and k can be determined from the tire-work estimate plus the damage energy of vehicle pairs k-1,k and k, k+1:

$$E_{abs,n-1}\left(\frac{\sum_{i=1}^{k-1} M_i \ (M_{k+1})}{\sum_{i=1}^{k+1} M_i \ (M_k)} + 1\right) = \frac{1}{2} K_{k,k+1} X^2_{k,k+1|n} + \frac{1}{2} K_{k-1,k} X^2_{k-1,k|n} + \frac{M_{k+1}}{2\sum_{i=1}^{k+1} M_i} W_{tires,n} \quad \text{(XXIX)}$$

Equation (X) can be applied to express the absorbed energy in collision n-1 in terms of the impact speed in that collision (substitute k-1 for k, and n-1 for n). The impact speed in collision n-1 then equals:

$$V^2_{pre,n-1} = \left(\frac{\sum_{i=1}^{k} M_i \sum_{i=1}^{k+1} M_i}{\sum_{i=1}^{k-1} M_i \ (\sum_{i=1}^{k+1} M_i M_k + \sum_{i=1}^{k-1} M_i M_{k+1})}\right) * \left(K_{k,k+1} X^2_{k,k+1|n} + K_{k-1,k} X^2_{k-1,k|n} + \frac{M_{k+1}}{\sum_{i=1}^{k+1} M_i} W_{tires,n}\right) \quad \text{(XX)}$$

Finally, equations (XXVI) give an expression for the delta-V of vehicles k-1 and k in their initial impact.